Using a Transactor/Revolver Scorecard to Make Credit and Pricing Decisions

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The Standard Approach

All credit card applicants

Use a credit scorecard to make credit decision

Accepted credit card users

Transactors

Revolvers
Transactors and Revolvers

**Transactors**
- all are Goods
- profit from merchant service charge only

**Revolvers**
- higher chance to be Bads
- main profit from interest on the balance

- Important in terms of default risk
- More important in terms of profitability
- Could we estimate this when making credit decision?
Our Proposed Approach

All credit card applicants

- Use a credit scorecard
- Use a tran/rev scorecard

A combined score

Accepted credit card users
A Combined Score

Define: T-Transactor, R-Revolver, G-Good, B-Bad

A score gives the probability the new customer is likely to be Good:

\[ P(G|x) = P(T|x)P(G|x, T) + P(R|x)P(G|x, R) \]

Since no Transactor can default, \( P(G|x, T) = 1 \).
Therefore,

\[ P(G|x) = P(T|x) + P(R|x)P(G|x, R) \]
To Develop The Two Scorecards

The Tran/Rev Scorecard

• Using all data

\[ s_t(x) = \ln \left( \frac{P(T|x)}{P(R|x)} \right) \Rightarrow P(T|x) = \frac{1}{1 + e^{-s_t(x)}} , \quad P(R|x) = \frac{1}{1 + e^{s_t(x)}} \]

The Good/Bad Scorecard to Revolvers Only

• Using revolvers’ data only

\[ s_R(x) = \ln \left( \frac{P(G|x,R)}{P(B|x,R)} \right) \Rightarrow P(G|x,R) = \frac{1}{1 + e^{-s_R(x)}}, \quad P(B|x,R) = \frac{1}{1 + e^{s_R(x)}} \]
A Numeric Example

- Credit card data from a Hong Kong bank
- Accounts opened 2002-2005; Outcome period: 2006
- Total 6,308 accounts: 1,577 Bad, 4,731 Good
- List of variables: Occupation, Education type, Citizenship, Residential type, Employment status, Annual income, Months with bank and Age
- Use weight-of-evidence for all characteristics
- Use stepwise logistic regression
- Use ten-fold cross validation

<table>
<thead>
<tr>
<th></th>
<th>Good</th>
<th>Bad</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transactor(count and column %)</td>
<td>2958 (63%)</td>
<td>0 (0%)</td>
<td>2958 (47%)</td>
</tr>
<tr>
<td>Revolver(count and column %)</td>
<td>1773 (37%)</td>
<td>1577 (100%)</td>
<td>3350 (53%)</td>
</tr>
<tr>
<td>Total(count)</td>
<td>4731</td>
<td>1577</td>
<td>6308</td>
</tr>
</tbody>
</table>
## Coefficients for All Scorecards

<table>
<thead>
<tr>
<th>Variable (WoE)</th>
<th>Standard Scorecard (Event=Good)</th>
<th>Transactor/Revolver Scorecard (Event=Transactor)</th>
<th>Good/Bad Scorecard (by Revolvers only) (Event=Good)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient (Mean)</td>
<td>Coefficient (S.D.)</td>
<td>Coefficient (Mean)</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.3929***</td>
<td>0.0072</td>
<td>-0.1254**</td>
</tr>
<tr>
<td>Occupation</td>
<td>0.7796***</td>
<td>0.0292</td>
<td>0.5994***</td>
</tr>
<tr>
<td>Education type</td>
<td>1.3961***</td>
<td>0.0678</td>
<td>0.4701**</td>
</tr>
<tr>
<td>Citizenship</td>
<td>1.2286***</td>
<td>0.0743</td>
<td>0.9286***</td>
</tr>
<tr>
<td>Residential type</td>
<td>1.1147***</td>
<td>0.0578</td>
<td>0.6864***</td>
</tr>
<tr>
<td>Employment status</td>
<td>0.1146**#</td>
<td>0.1848</td>
<td>0.3951**</td>
</tr>
<tr>
<td>Months with bank</td>
<td>1.1741***</td>
<td>0.0187</td>
<td>0.7998***</td>
</tr>
<tr>
<td>Annual income</td>
<td>-</td>
<td>-</td>
<td>0.2150**</td>
</tr>
<tr>
<td>Age</td>
<td>-</td>
<td>-</td>
<td>0.2660**</td>
</tr>
</tbody>
</table>

*** significant at 0.0001; ** significant at 0.05; #selected by three models only. For models do not pick up the variable, we assume the coefficients equal 0.
# Gini Coefficients

<table>
<thead>
<tr>
<th>Validation</th>
<th>Standard</th>
<th>Tran/Rev</th>
<th>G/B with Rev only</th>
<th>Combined</th>
<th>ROC Contrast Test Results between Standard and Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.592</td>
<td>0.45</td>
<td>0.596</td>
<td>0.592</td>
<td>0.039(0.8434)</td>
</tr>
<tr>
<td>2</td>
<td>0.47</td>
<td>0.394</td>
<td>0.47</td>
<td>0.474</td>
<td>0.5008(0.4791)</td>
</tr>
<tr>
<td>3</td>
<td>0.556</td>
<td>0.45</td>
<td>0.542</td>
<td>0.558</td>
<td>0.0488(0.8251)</td>
</tr>
<tr>
<td>4</td>
<td>0.52</td>
<td>0.42</td>
<td>0.518</td>
<td>0.511</td>
<td>2.1165(0.1457)</td>
</tr>
<tr>
<td>5</td>
<td>0.482</td>
<td>0.426</td>
<td>0.486</td>
<td>0.48</td>
<td>0.0789(0.7788)</td>
</tr>
<tr>
<td>6</td>
<td>0.526</td>
<td>0.4</td>
<td>0.53</td>
<td>0.53</td>
<td>0.3427(0.5583)</td>
</tr>
<tr>
<td>7</td>
<td>0.512</td>
<td>0.428</td>
<td>0.506</td>
<td>0.508</td>
<td>0.9092(0.3403)</td>
</tr>
<tr>
<td>8</td>
<td>0.524</td>
<td>0.44</td>
<td>0.522</td>
<td>0.532</td>
<td>0.057(0.8114)</td>
</tr>
<tr>
<td>9</td>
<td>0.548</td>
<td>0.432</td>
<td>0.54</td>
<td>0.542</td>
<td>0.8449(0.358)</td>
</tr>
<tr>
<td>10</td>
<td>0.484</td>
<td>0.47</td>
<td>0.48</td>
<td>0.49</td>
<td>1.2506(0.2634)</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>0.522</strong></td>
<td><strong>0.431</strong></td>
<td><strong>0.519</strong></td>
<td><strong>0.522</strong></td>
<td></td>
</tr>
</tbody>
</table>
ROC Curves for Two Folds

**Validation 1**

ROC Curves for Comparisons

- ROC Curve (Area)
  - Model 1 (0.7959)
  - Model 4 (0.7964)

**Validation 4**

ROC Curves for Comparisons

- ROC Curve (Area)
  - Model 1 (0.7597)
  - Model 4 (0.7557)
Credit Card Profitability Model

If the interest rate offered on credit cards is $r$, the corresponding expected monthly profit for the lender is:

$$E(r) = \int_{p^*(r)}^{1} e(r, p) q(r, p) f(p) dp$$

- $e(r, p)$: the expected monthly profit of a customer with hazard rate $p$
- $f(p)$: The distribution of the Good hazard rates
- $p^*(r)$: the cut-off level of the hazard rate of being Good
- $q(r, p)$: a customer’s take-up probability
Risk and Take Function

- Population’s hazard risk distribution

\[ F(p) = \begin{cases} 
0, & p < 0.5 \\
2p^2 - 2p + 0.5, & 0.5 \leq p < 1 \\
1, & p = 1 
\end{cases} \]

- Take Function (Phillips, 2005; Thomas, 2009)

\[ q(r, p) = \text{Max}\{0, 3 - 10r - 2p\} \]

- For example, if interest rate \( r = 3\% \) and the hazard rate is 0.9, the take rate is

\[ q(3\%, 0.9) = 3 - 10(3\%) - 2(0.9) = 90\% \]
The Expected Profit: $e(r, p)$

- Income – interchange fees and interest on balance
- The expected profit:

\[
e(r, p) = \text{Interchange fee} - \text{Average Purchase} \\
+ P(\text{Not Default in N period}) \times (\text{Repayment in N period}) \\
+ P(\text{Default in N period}) \times (\text{Recovery via Collection})
\]

- Given an interest rate, find the optimal cut-off probability by

\[
e(r, p^*) = 0 \Rightarrow p^* = \left(\frac{(1 - m)(1 + r_F)^N}{l_D (1 + r)^{N-1}} + \frac{l_D - 1}{l_D}\right)^{1/N}
\]

Parameters required

- $m$ : Interchange rate
- $r$ : Interest per period of the credit cards
- $r_F$ : Interest rate at which lenders can borrow money per period
- $l_D$ : Loss given default (LGD)
- $N$ : Average number of periods before repaying the average purchase
To Estimate $N$

$N$: average number of periods before a purchase is paid off

$B$: average balance carried over per period per customer

$P$: average amount purchased per period per customer

$C$: average repayment amount per period per customer

Interest paid + Ave. Expenditure = Ave. Repayment,
i.e.

$$rB + P = C \quad \text{........................................... \text{(*)}}$$
To Estimate $N$ (cont.)

- Assume the user pays off the costs in the order they are incurred

\[ (1 + r)B + P = CN \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (***) \]

- Using (*) and (**),

\[
B = \frac{C - P}{r}
\]

\[
N = \frac{(1 + r)B + P}{C} = \frac{B + C}{C}
\]
A Numerical Example

<table>
<thead>
<tr>
<th>$m$</th>
<th>Interchange rate</th>
<th>2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_F$</td>
<td>Interest rate at which lender can borrow money per period</td>
<td>1%</td>
</tr>
<tr>
<td>$l_D$</td>
<td>LGD</td>
<td>60%</td>
</tr>
<tr>
<td>$P$</td>
<td>Average purchase per period</td>
<td>51</td>
</tr>
<tr>
<td>$C$</td>
<td>Average repayment per period</td>
<td>60</td>
</tr>
</tbody>
</table>

Using the above parameters and the equations listed before, we can find the expected profit and the corresponding hazard rate:

<table>
<thead>
<tr>
<th>$r$</th>
<th>$p^*$</th>
<th>$(p^*)^{12}$</th>
<th>$E(r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3%</td>
<td>0.969</td>
<td>0.687</td>
<td>2.084</td>
</tr>
<tr>
<td>2%</td>
<td>0.983</td>
<td>0.817</td>
<td>1.783</td>
</tr>
<tr>
<td>4%</td>
<td>0.957</td>
<td>0.590</td>
<td>2.040</td>
</tr>
</tbody>
</table>
If the interest rate offered on credit cards is \( r \), the corresponding expected monthly profit for the lender is:

\[
E(r) = \int_{-\infty}^{\infty} dt \int_{p^*_R(t)}^{1} e(r, p) q(r, p) f(p, t) dp
\]
Risk and Take Function

• Same take function

\[ q(r, p) = \text{Max}\{0, 3 - 10r - 2p\} \]

• Population’s hazard risk distribution

\[
F(p, t) = \begin{cases} 
0 & p < 0.5 \\
2p^2 - 2p + 0.5 & 0.5 \leq p < 1, 2p - 1 > t \\
2tp - t - 0.5t^2 & 0.5 \leq p < 1, 2p - 1 \leq t \\
1 & p = 1, t = 1
\end{cases}
\]

Percentage of transactors

\[
\bar{t} = \int_{0}^{1} \int_{0}^{1} f(p, t) \, dp = \frac{2}{3}
\]
The Expected Profit: $e(p_R, t)$

- $t$: the transactor score
- $p_R$: the Good hazard rate from the Revolver Good/Bad scorecard
- $e(p_R, t) = t \times (\text{Interchange fee})$
  
  $$+(1 - t)[\text{Interchange fee} - \text{Average Purchase}$$
  
  $$+ P(\text{Not Default in N period}) \times (\text{Repayment in N period})$$
  
  $$+ P(\text{Default in N period}) \times (\text{Recovery via Collection})]$$

- Given an interest rate, find the optimal cut-off probability by
  
  $$e(p_R^*, t) = 0$$

  $$\Rightarrow p_R^* = \left(\frac{(1 + r_F)}{l_D(1 + r)^{N_R-1}} \left(\frac{tP_T}{(1 - t)P_T} \left(1 - m - \frac{1}{1 + r_F}\right) + 1 - m\right) + \frac{l_D - 1}{l_D}\right)^{1/N_R}$$
With a Tran/Rev Scorecard

\( N \): average number of periods before a purchase is paid off
\( B \): average balance carried over per period per customer
\( P \): average amount purchased per period per customer
\( C \): average repayment amount per period per customer

**Revolvers**

\[
rb_R + P_R = C_R
\]
\[
(1 + r)b_R + P_R = C_RN_R
\]

**Transactors**

\[
B_T = 0
\]
\[
C_T = P_T \Rightarrow N_T = 1
\]

**All users**

\[
P = P(T)P_T + (1 - P(T))P_R,
\]
\[
C = P(T)C_T + (1 - P(T))C_R,
\]
\[
B = (1 - P(T))b_R,
\]
\[
N = \alpha N_R + (1 - \alpha) \text{ where } \alpha = \frac{C_R(1 - P(T))}{C_T P(T) + C_R(1 - P(T))}
\]
The Numerical Example with Tran/Rev Split

<table>
<thead>
<tr>
<th>( P )</th>
<th>Average purchase per period</th>
<th>51</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>Average repayment per period</td>
<td>60</td>
</tr>
<tr>
<td>( P(T) )</td>
<td>Percentage of transactors</td>
<td>2/3</td>
</tr>
</tbody>
</table>

\[
p^*_R(t): \text{the optimal cut-off corresponding to transactor score } t
\]

<table>
<thead>
<tr>
<th>( r )</th>
<th>( t )</th>
<th>( E(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3% )</td>
<td>0</td>
<td>0.960</td>
</tr>
<tr>
<td>( 4% )</td>
<td>0</td>
<td>0.924</td>
</tr>
<tr>
<td>( 2% )</td>
<td>0</td>
<td>0.982</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.959</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.956</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.953</td>
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<tr>
<td></td>
<td>0.5</td>
<td>0.950</td>
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<tr>
<td></td>
<td>0.6</td>
<td>0.943</td>
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<tr>
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<td>0.7</td>
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<td>0.000</td>
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<td>0.000</td>
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<td></td>
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<td>0.399</td>
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<tr>
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<td>0.981</td>
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<tr>
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<td>0.980</td>
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<tr>
<td></td>
<td>0.3</td>
<td>0.979</td>
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<tr>
<td></td>
<td>0.4</td>
<td>0.977</td>
</tr>
<tr>
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<td>0.5</td>
<td>0.975</td>
</tr>
<tr>
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<td>0.6</td>
<td>0.971</td>
</tr>
<tr>
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<td>0.7</td>
<td>0.963</td>
</tr>
<tr>
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<td>0.940</td>
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<td>0.9</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.311</td>
</tr>
</tbody>
</table>
Conclusion & Possible Extensions

• Build a scorecard to estimate $P(T|x)$
• How to use the score in profitability modelling
• The model with Tran/Rev: acknowledge the profitability of Transactors so that the estimation on profitability is more accurate and offer a different price
• Use it for Churn prediction?
Thank you