The Extended Exogenous Maturity Vintage Model Across the Consumer Credit Lifecycle

Malwandla, M. C.¹,²
Rajaratnam, K.³
Clark, A. E.¹

1. Department of Statistical Sciences, University of Cape Town, Cape Town, South Africa
2. Standard Bank, Johannesburg, South Africa
3. Department of Finance & Tax and the African Collaboration for Quantitative Finance and Risk Research, University of Cape Town, Cape Town, South Africa
4. African Collaborative for Quantitative Finance and Risk Research, Cape Town, South Africa

Presented By: Musa Malwandla
Research Problem and Aim

- Identifiability Problem in the EMV Model, and lack of behavioural data
  - Aim: bypass the identifiability problem
  - Aim: introduce behavioural data into model

- Credit risk models lack unification
  - Aim: show how model can be used across all areas of credit risk

- Lack micro-foundation for aggregation model, understanding of systemic risk
  - Aim: show how model aggregates to portfolio loss
  - Aim: provide general formula for asset correlation coefficient

- The problem of aggregating risk across portfolios
  - Aim: provide an approach for aggregating risk across portfolios
Agenda

- **Model Specification**
  - Standard EMV Model
  - Extended EMV Model
  - Illustration: South African Portfolio

- **Application Areas**
  - Application and Behavioural Scorecard
  - Impairment Modelling
  - Stress Testing
  - Capital Management

- **Extension: Survival Analysis Decomposition**

- **Systemic Risk & Aggregation**
  - Understanding the LHP approximation
  - Understanding the Asset Correlation Coefficient
  - General Asset Correlation Coefficient Formula
  - Illustration: South African Portfolio
  - Understanding Diversifiable

- **Cross-Portfolio Aggregation**
  - A formula for cross-portfolio aggregation
  - Other uses of the formula
  - Illustration: South African Portfolio
Model Specification

- Standard EMV Model
- Extended EMV Model
- Illustration: South African Portfolio
Standard EMV Model

APC Model
- Widely applied in epidemiology: Age Period Cohort Model
- Dimensional interpretation in mortality studies:
  - **Age**: effect of age on mortality (e.g., Gompertz Law, Accident Hump)
  - **Period**: effect of period on mortality (e.g., war, plague, cultural influence)
  - **Cohort**: effect of cohort on mortality (e.g., epigenetics, unnatural selection)

EMV Model
- **Exogenous**: mainly assumed to capture macroeconomic/policy environment
- **Maturity**: mainly captures risk by maturity and selective effects
- **Vintage**: mainly captures effect of acquisition policy (e.g. scorecard and cut-offs, target market)

\[
p(t,s) = A_{t-s} + P_t + C_s
\]
\[
p(t,s) = M_{t-s} + E_t + V_s
\]
Extended EMV Model: Specification

\[ p(t, s, k) = \Phi(\alpha + M_{t-s} + E_t + \Psi_s + B_k + A_k) \rightarrow \Phi(\mu + \sigma_M \tilde{M}_{t-s} + \sigma_E \tilde{E}_t + \sigma_V \tilde{\Psi}_s + \sigma_B \tilde{B}_k + \sigma_B \tilde{A}_k) \]

- **Extend model beyond time dimensions, to behavioural dimensions**
  - Include behavioural score dimension / application score dimension...

- **Standardisation**
  - For convenience, standardise all component to mean = 0, std. dev = 1
  - Standard deviation of each component becomes parameter / significance measure

- **Identifiability**
  - Application scorecard attempts to capture same effect as vintage
  - Replace vintage with application scorecard

- **Link functions**
  - Logit, Probit, CLogLog
  - Prefer Probit: leads to Vasicek distribution for portfolio loss
  - Alternative is CLogLog: leads to Log-Log-Normal distribution for portfolio loss
Fitting Illustration: Portfolio Description

- South African consumer loan portfolio
  - **Fixed-Rate Loans**: interest rate fixed at the outset of the loan
  - **Variable-Rate Loans**: interest rate varies with central bank rate (Prime Overdraft Rate)

- Observations
  - 2.5m observations
  - September 2005 to June 2014

- Default Definition:
  - 90 Days Past Due, Distressed Restructure, Litigation, Write Off
### Fitting Illustration: Dimensionality

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Fitting Illustration: Components

\[ p(t, s, k) = \Phi(\mu + \sigma_M \tilde{M}_{t-s} + \sigma_E \tilde{E}_t + \sigma_B \tilde{B}_k) \]
Fitting Illustration: Exogenous Model

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<th>P-Value</th>
<th>VIF</th>
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### Fitting Illustration: Validation

**Gini Statistic**

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<td><strong>Fixed Rate Loans</strong></td>
<td>37%</td>
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<td><strong>Variable Rate Loans</strong></td>
<td>47%</td>
<td>14%</td>
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**Accuracy Across Range**

Accuracy across range from 0% to 30%.

**Accuracy Across Time**

Accuracy across time from 2005.01 to 2014.05.
Application Areas

Application and Behavioural Scorecard
Impairment Modelling
Stress Testing
Capital Management
Application Areas: Scorecards

- Linking risk scores to macroeconomic variables:
  - Dynamic score cut-offs
  - Reducing recalibration frequency

IFRS 9 SICR swap-set problem can be resolved through EMBA model:

- \( p(t, s, k) = \Phi(E_t + R_{t-s, l, k}) \)
  where
  \( R_{t,s,l,k} = V_t \alpha_{t-s} + B_k (1 - \alpha_{t-s}) \)
  for monotonic \( \alpha_{t-s} \)
Application Areas: Impairments & Stress-Testing

- IFRS 9 Lifetime PD
  - Use Cumulative Incidence Function approach:
    \[ p_k(t, R_t, s, l, k) = \sum_{j=1}^{k} p_{12}(t, R_{t+j, s, l, k}) \times \left[ 1 - p_{12}(t, R_{t+j-1, s, l, k}) - q_{12}(t, R_{t+j-1, s, l, k}) \right] \]

- Stress-Testing
  - Acquisition strategy using application score component.
  - Account management strategy, through behavioural score card.
Application Areas: Capital Management (Model Error)

Random Effect or Model Error

**Full Error** $= \sigma_E^2$
**Residual Error** $= (1 - r^2)\sigma_E^2$
## Extended EMV Model: Summary

<table>
<thead>
<tr>
<th>Application Area</th>
<th>Modelled Parameter</th>
<th>Point-in-Time / Forward-in-Time</th>
<th>Through-the-Cycle</th>
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<tbody>
<tr>
<td>Application / Behavioural Scoring / Stress Testing</td>
<td>Probability of Default; of Rolling (Collections); Write Off (LGD)</td>
<td>$\phi \left( \frac{R_{t,s,l,k}}{\sqrt{1 + (1 - r^2)\sigma_E^2}} \right)$</td>
<td>$\phi \left( \frac{K_{t,s,l,k}}{\sqrt{1 + \sigma_E^2}} \right)$</td>
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<tr>
<td>IFRS 9 Impairment Modelling</td>
<td>Lifetime Probability of Default</td>
<td>Cumulative Incidence Function</td>
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<td>Economic Capital</td>
<td>Value-at-Risk / Quantile Function</td>
<td>$\phi \left[ \sqrt{\frac{\rho}{1 - \rho}} \phi^{-1}(\alpha) + \sqrt{\frac{1}{1 - \rho}} \phi^{-1}(p_t) \right]$</td>
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<tr>
<td>Economic Capital</td>
<td>Asset Correlation Coefficient</td>
<td>$\frac{\sigma_E^2(1 - r^2)}{1 + \sigma_E^2(1 - r^2)}$</td>
<td>$\frac{\sigma_E^2}{1 + \sigma_E^2}$</td>
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<tr>
<td>Economic Capital (Cross-Portfolio)</td>
<td>Cross-Portfolio Loss Distribution</td>
<td>$N(\mu, \sigma)$</td>
<td>$N(\mu, \sigma)$</td>
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</table>

### Short hand:
- $R_{t,s,l,k} = \mu + \sigma_M \tilde{M}_{t-s} + \sigma_V \tilde{V}_l + \sigma_B \tilde{B}_t + \sigma_E \tilde{E}_t$
- $K_{t,s,l,k} = \mu + \sigma_M \tilde{M}_{t-s} + \sigma_V \tilde{V}_l + \sigma_B \tilde{B}_k$
Systemic Risk & Aggregation

Understanding the LHP approximation
Understanding the Asset Correlation Coefficient
General Asset Correlation Coefficient Formula
Illustration: South African Portfolio
Understanding Diversifiable
Understanding the LHP Approximation

- Condition on $e$
- Assume homogeneity

Unknown distribution
Unknown distribution

Vasicek Quantile Function

$$
\Phi \left[ \frac{\rho}{1 - \rho} \phi^{-1}(\alpha) + \sqrt{\frac{1}{1 - \rho}} \phi^{-1}(p_t) \right]
$$

FYI: CLogLog link leads to log-log-normal distribution

Un-condition on $e$, Assume Gaussian $e$ and Probit link function

Vasicek Distribution

Binomial distribution

Assume large portfolio

Degenerate distribution

Unknown distribution
Understanding the Asset Correlation Coefficient

The asset correlation coefficient is the shape measure of the Vasicek distribution:
- Influenced by level of correlation between portfolio, or
- Influenced by level systemic risk relative to idiosyncratic risk

![Asset Correlation Diagrams](image-url)
General Asset Correlation Coefficient Formula

The asset correlation coefficient is influenced by:
- Level of risk within the system/economy $\sigma_S^2$
- Pro-cyclicality of portfolio $\beta$
- Level of portfolio’s systemic risk that is explained by the model $r^2$

To get TTC confidence intervals, simply set $r^s = 0$

The formula supersedes the generic formulae offered under Basel.
Illustration of Value-at-Risk

Point-in-Time PD Confidence Interval: Fixed Rate Loans

Through-the-Cycle PD Confidence Interval: Fixed Rate Loans

Point-in-Time PD Confidence Interval: Variable Rate Loans

Through-the-Cycle PD Confidence Interval: Variable Rate Loans
Understanding (Non-)Diversifiable Risk

- For homogenous population, diversifiable risk follows a scaled binomial distribution.
- Due to the asymptotic aspect of LHP assumption, the loss distribution ignores sampling error (diversifiable risk).
- However, observed PD subject to sampling error, which filters through to exogenous component.
- Therefore, the estimate asset correlation coefficient captures total risk.
Cross-Portfolio Aggregation

A formula for cross-portfolio aggregation
Other uses of the formula
Illustration: South African Portfolio
Formula for Cross-Portfolio Aggregation

- **Aim**: to allow for diversification benefits across portfolios

- **Assumptions**:
  - Assume exogenous error size is small...
  - So that Taylor approximation applies: 
    \[ p_k(e_{k,t}) \approx \phi(R_{k,t}) + \phi'(R_{k,t})\sigma_{k,e} e_{k,t} \]
  - Assume that portfolio errors normally distributed \( e_{k,t} \) and linearly correlate
  - Joint loss distribution becomes multivariate normal

- **Practical uses**:
  - Working out required capital for an entire bank
  - Capital budgeting: Determining how much a portfolio contributes to the total risk
  - System analysis: Determining how much a bank contributes to systemic risk
Illustration

Combined PD Confidence Interval: Point-in-Time

Combined PD Confidence Interval: Through-the-Cycle
**Final Comments on Error Distribution**

- Most talk about error distribution looks at the fatness of the tails (Kurtosis)
  - i.e., attempt to capture extreme events.

- This presentation suggests that:
  - For poor-fitting macroeconomic models, autocorrelation matters.
  - For small portfolios with low default rates, skewness matters.
  - For aggregating across portfolio, the joint error distribution matters.
Conclusion

- **Extended EMV Model**
  - Solving (bypassing) the identifiability problem
  - Survival analysis decomposition

- **Unification**
  - Application in scoring, impairment, stress testing, capital management

- **Systemic Risk**
  - Understanding loss aggregation and asset correlation
  - General formula for asset correlation

- **Cross-Portfolio Aggregation**
  - Aggregation across portfolio

- **Further Research**
  - Error distribution, and joint error distribution
  - Combining model with other risk types (e.g., liquidity, operational)
Appendix

Application of EMV-type decomposition to Survival Analysis
Extension: Survival Analysis Decomposition

\[ h_{j,s}(t) = \Phi \left( \alpha_1 b_t + \alpha_2 \varphi_{j,s} + \alpha_3 e_s \right) \] or \[ h_{j,s}(t) = \Phi(\alpha_1 b_t + \alpha_3 e_s) e^{\varphi_{j,s}} \equiv b_t e^{\varphi_{j,s}} \]

**Objective:**
- Survival analysis with time varying covariates
- Non-parametric estimation without need for partial-likelihood function
Extension: Survival Analysis Decomposition
Extension: Survival Analysis Decomposition

PD by Horizon: Cycle 0

PD by Calendar Month: Cycle 0

Accuracy Across Range: Cycle 0, Horizon 12

Accuracy Across Range: Cycle 0, Horizon 36
## EMV Survival Analysis Data Setup

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Questions

Please contact me at: malwandla@live.co.za